

Is Bohm's interpretation of quantum mechanics consistent?

Abstract

The supposed equivalence of the conventional interpretation of quantum mechanics with Bohm's interpretation is generally demonstrated only in the coordinate representation. It is shown, however, that in the momentum representation this equivalence is not valid.

Recently, there has been a renewed interest in David Bohm's interpretation of *non-relativistic* quantum mechanics (QM) [1]-[3] and many pedagogical papers on this topic have appeared[4]-[15], while on line, arXiv.org lists over 200 submissions on this topic during the past ten years. Bohm claimed that “as long as the present general form of Schrödinger's equation is retained the physical results obtained with this suggested alternative are precisely the same as those obtained with the conventional interpretation”, and that his interpretation “leads in all possible experiments to the same predictions as are obtained from the usual interpretation” [1]. Similar assertions also have been made in references [4]-[15], but this equivalence is usually demonstrated only in the coordinate representation, while the implications of Bohm's interpretation in the momentum representation are usually ignored. While there have been some criticisms in the past of Bohm's interpretation of QM, (for example, see reference [16],[17]), we give here an elementary proof that the momentum distribution in this interpretation differs from that in standard QM. We show that the definition of particle velocity in this interpretation, implies that the product of mass times velocity is not equal to momentum, which is inconsistency with both classical and quantum mechanics.

In Bohm's interpretation of quantum mechanics, the velocity of a particle with mass m is given by

$$\vec{v}_B = \vec{\nabla} S / m \quad (1)$$

where S/\hbar is the phase of the wave function ψ obtained by solving the time dependent Schrödinger equation. According to Bohm,

$$\vec{v}_B = \frac{d\vec{q}}{dt}, \quad (2)$$

where \vec{q} is the time dependent coordinate for the position of the particle, and Eq.1 becomes a first order differential equation that determines \vec{q} as a function of time t , given its initial value [18]. But it turns out that the product $m\vec{v}_B$ is not equal to the canonical momentum \vec{p} , because \vec{v}_B does not correspond to the velocity \vec{v} , that is determined in quantum mechanics by the operator

$$\vec{v} = -\frac{i\hbar}{m}\vec{\nabla}_q = \frac{\vec{p}}{m}. \quad (3)$$

A proof of this relation is given in Appendix A. Setting

$$\psi = R \exp(iS/\hbar), \quad (4)$$

where R is the amplitude of ψ , we obtain

$$\vec{v}\psi = (\vec{\nabla}_q S/m - i\hbar\vec{\nabla}_q R/mR)\psi. \quad (5)$$

But in Bohm's definition of the particle velocity, Eq. 1, only the first term on the right hand side of this equation appears. The relevance of the second term can be illustrated by considering the mean values $\langle \vec{v} \rangle$ and $\langle \vec{v}^2 \rangle$ in this representation for ψ . We have

$$\langle \vec{v} \rangle = \int d^3q \psi^\dagger \vec{v} \psi = \int d^3q R^2 \vec{\nabla} S/m = \langle \vec{v}_B \rangle, \quad (6)$$

and

$$\langle \vec{v}^2 \rangle = \int d^3q \psi^\dagger (\vec{v})^2 \psi = \langle (\vec{v}_B)^2 \rangle + (\hbar/m)^2 \int d^3q (\vec{\nabla} R)^2. \quad (7)$$

Hence, Eq.7 implies that the second moment of the velocity distribution in conventional quantum mechanics differs from that obtained in Bohm's interpretation of the particle velocity, Eq. 1, by the appearance of the additional term $(\hbar/m)^2 \langle (\vec{\nabla} R)^2/R^2 \rangle$ on the right hand side of this equation. Remarkably, this discrepancy [19] is not even mentioned in any of the recent articles on Bohm's interpretation of wave mechanics [4]-[14].

To get agreement with the mean value $\langle \vec{v}^2 \rangle$ in quantum mechanics, Eq.7, Bohm's interpretation requires, in addition to the Bohmian particle velocity \vec{v}_B given by Eq.1, the existence of an *ad hoc* random velocity

$$\vec{v}_o = \frac{\hbar}{mR} \vec{\nabla} R, \quad (8)$$

with vanishing mean value. Originally, such a contribution was introduced with an undetermined coefficient as a *random* velocity by D. Bohm and J. P. Vigier [20], who named it an "osmotic velocity", after a term introduced by Einstein to describe the chaotic Brownian motion. But now such a term has been abandoned in discussions of Bohmian mechanics.

In particular, for stationary solutions of the Schrödinger, the phase $S = 0$, and Bohm's interpretation leads to the conclusion that the particle velocity vanishes in such a state. This conclusion is explained by invoking a *quantum force* due to a non-local *quantum potential* that supposedly balances the force due to the conventional potential that gives rise to the stationary solution. This non-classical force appears when the acceleration $d^2\vec{q}/dt^2$ is calculated by taking the time derivative of Eqs. 1 and 2. But this result contradicts the fact that in quantum mechanics the velocity or momentum distribution for stationary solutions, given by the absolute square of the Fourier transform of ψ in coordinate space, is not a delta function at $\vec{v} = 0$, as is implied by Bohm's interpretation.

The trajectories obtained by integrating Bohm's first order differential equation for the particle coordinate \vec{q} , Eq. 2, correspond to *pathlines* associated with the probability distribution $\rho = |\psi|^2$ which satisfies, like a normal fluid of density ρ , the continuity equation,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_q \cdot \vec{j} = 0 \quad (9)$$

where $\vec{j} = \vec{v}_B \rho$ is the associated current. While pathlines provide a visualization of a fluid flow, these lines do not correspond to the actual motion of the particles composing the fluid that also can have a random component. Likewise, Bohmian pathlines serve to visualize the evolution of the probability distribution in quantum mechanics, but do not correspond to actual trajectories of elementary particles.

Recently, experiments have been made with water droplets surfing on the waves produced by the Faraday instability on the surface of an oscillating

tank filled with a fluid [21]. The motion of these droplets mimics the suggestion of de Broigle and of Bohm that elementary particles are likewise “piloted” by the ψ function of wave mechanics. In particular, it is claimed that when the waves propagate through two slits, or are confined in a “corral”, the droplets satisfy statistics that are similar to those observed for particles in quantum mechanics [22]. But such experiments only demonstrate the universality of wave propagation, and the associated pathlines, whether governed by the equations of fluid mechanics, quantum mechanics, or of other sources of waves in physics.

Appendix A. The relation between velocity and momentum in non-relativist quantum mechanics

In quantum mechanics, the velocity \vec{v} , like the position \vec{q} and the momentum \vec{p} , is an operator. It is *defined* by the relation

$$\vec{v} = \frac{i}{\hbar} [H, \vec{q}], \quad (10)$$

where H is the hamiltonian operator, and $[a, b] = ab - ba$ is the commutator of the operators a and b . In non-relativistic quantum mechanics,

$$H = -\frac{\hbar^2}{2m} \nabla_q^2 + V(\vec{q}), \quad (11)$$

corresponding to the time dependent Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad (12)$$

Hence, substituting this expression for H in Eq.10, one finds that the velocity operator is given by

$$\vec{v} = \frac{\vec{p}}{m} \quad (13)$$

where

$$\vec{p} = -i\hbar \vec{\nabla}_q \quad (14)$$

is the momentum operator.

For an alternative derivation of the connection between the velocity and momentum operators, Eq.13, that does not presuppose the Schrödinger equation, Eqs.11 and 12, consider the commutation relation Eq.10 for the Hamiltonian of a free particle $H_o = \vec{p}^2/2m$. Then, according to the definition of velocity, Eq.10,

$$v_i = \frac{i}{2\hbar m}(p_j[p_j, q_i] + [p_j, q_i]p_j), \quad (15)$$

and substituting the Heisenberg-Born commutation relation

$$[p_j, q_i] = -i\hbar\delta_{i,j} \quad (16)$$

leads again to Eq. 13.

References

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<http://plato.stanford.edu/entries/qm-bohm/>. In reference [10], J. Bernstein also asserted that “when unambiguous the predictions of the two theories are identical.”

- [16] A related demonstration of the inconsistency of Bohm’s interpretation of quantum mechanics was pointed out already 59 years ago in J.B. Keller, “Bohm’s Interpretation of the Quantum Theory in terms of ‘Hidden Variables’,” *Phys. Rev.* **89**, 1040-1041 (1953), but it has either been ignored or forgotten.
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- [18] In his original papers, [1], Bohm introduced as fundamental, the equation of motion for the acceleration $d\vec{v}/dt$. This equation can be obtained by taking the time derivative of Eq. 1, but it is misleading to regard it as *fundamental*, because it implies that the initial velocity of the particle can be assigned arbitrarily. But given the initial position \vec{q} , this velocity is determined uniquely by Eq. 1. Bohm’s equation of motion leads to the appearance of a non-local “quantum potential” that accounts for the origin of an acceleration even when the classical potential vanishes.
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